

The $\mathfrak{sl}(\mathcal{N})$ twisted Yangian: bulk-boundary scattering & defects

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Abstract

We consider the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian quantum spin chain. In particular, we study the bulk and boundary scattering of the model via the solution of the Bethe ansatz equations in the thermodynamic limit. Local defects are also implemented in the model and the associated transmission amplitudes are derived through the relevant Bethe ansatz equations.

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1 Introduction

Spin chains can be considered as main paradigms of quantum integrable systems: the discrete structure on the lattice allows to disregard complications of infinite quantities arising from same-site manipulations of generators, and the characteristic structure of co-product (or alternatively co-module in the reflection case) of the underlying quantum algebras (Yangian, Twisted Yangian, Quantum Groups, Reflection algebras, Elliptic Algebras) directly translates into the site-by-site building of the space of quantum states and the Hamiltonians deduced from monodromy matrices.

Open spin chains, requiring introduction of boundary terms consistent with quantum integrability, are in particular related to generalized reflection algebras (quadratic algebras) à la Freidel-Maillet [1] extending the original construction of Cherednik [2] and Sklyanin [3] to a four matrix structure canonically expressed as:

$$A_{12} K_1 B_{12} K_2 = K_2 C_{12} K_1 D_{12}, \quad (1)$$

with unitarity requirements

$$\begin{aligned} A_{12} A_{21} &= D_{12} D_{21} = \mathbb{I}_{12}, \\ C_{12} &= B_{21}. \end{aligned} \quad (2)$$

In the particular case when $A_{12} = D_{21} = R_{12}$ a given Yang-Baxter R matrix, and $B_{12} = C_{21} = \bar{R}_{21}$ (its soliton anti-soliton counter part), $\bar{R}_{12} \sim R_{12}^{t_1}$, (1) yields the so-called twisted Yangian structure if R is the simple Yangian solution of the Yang-Baxter equation [4].

Spin chains based on such a twisted Yangian were first constructed and investigated in [6]. They were then considered in the thermodynamic limit in our previous paper [5]. They naturally exhibit soliton non-preserving boundary conditions due the choice of $B_{12} = C_{21}$ as a soliton–anti-soliton S -matrix and the subsequent conversion of a soliton into an anti-soliton by the building reflection matrix K .

We describe here the additional effect of a defect in such a spin chain for a general A_n underlying Lie algebra. Assuming that the spin chain is as usual derived from a monodromy matrix obtained from the canonical representation of the co-module algebra by the coefficient matrices A, C or D, B , defects can be naturally implemented in the context of spin chain model building by inserting a different representation of the co-module algebra into the double row transfer matrix at one physically meaningful site (the defect site): since we are dealing with open spin chains the defect is in fact inserted at two “mirror” imaged sites when building the monodromy matrix. Note that we consider here the case of a purely transmitting defect, the reflecting-transmitting defects require more intricate manipulations (see e.g. [7]).

The paper runs as follows: we recall the Bethe ansatz equations (BAE) for the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian spin chain and define the energy of particle-like excitations. We then derive formulas for both bulk and boundary scattering amplitudes. We finally address the issue of computing the particle-defect transmission amplitudes. The corresponding quantization condition is also derived.

2 BAE for $\mathfrak{sl}(\mathcal{N})$ twisted Yangian

The main aim of this section is the study of the Bethe ansatz equations in the thermodynamic limit. In particular, the ground state and the low lying excitations of the model are identified. The BAE for the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian were derived in [8]. We shall distinguish below two cases $\mathcal{N} = 2n$ or $\mathcal{N} = 2n + 1$. Note that throughout the text we consider the boundary matrices, c -number representations of the twisted Yangian (2) ($A_{12} = D_{21} = R_{12}$, and $B_{12} = C_{21} = \bar{R}_{21}$), to be proportional to unit. Defining

$$e_n(\lambda) = \frac{\lambda + \frac{in}{2}}{\lambda - \frac{in}{2}}, \quad (3)$$

the BAE read as follows:

- $\mathfrak{sl}(2n+1)$

$$\begin{aligned}
e_1^L(\lambda_i^{(1)}) &= - \prod_{j=1}^{M^{(1)}} e_2(\lambda_i^{(1)} - \lambda_j^{(1)}) e_2(\lambda_i^{(1)} + \lambda_j^{(1)}) \prod_{j=1}^{M^{(2)}} e_{-1}(\lambda_i^{(1)} - \lambda_j^{(2)}) e_{-1}(\lambda_i^{(1)} + \lambda_j^{(2)}), \\
1 &= - \prod_{j=1}^{M^{(\ell)}} e_2(\lambda_i^{(\ell)} - \lambda_j^{(\ell)}) e_2(\lambda_i^{(\ell)} + \lambda_j^{(\ell)}) \prod_{\tau=\pm 1} \prod_{j=1}^{M^{(\ell+\tau)}} e_{-1}(\lambda_i^{(\ell)} - \lambda_j^{(\ell+\tau)}) e_{-1}(\lambda_i^{(\ell)} + \lambda_j^{(\ell+\tau)}) \\
&\text{for } \ell = 2, \dots, n-1, \\
e_{-\frac{1}{2}}(\lambda_i^{(n)}) &= - \prod_{j=1}^{M^{(n)}} e_{-1}(\lambda_i^{(n)} - \lambda_j^{(n)}) e_{-1}(\lambda_i^{(n)} + \lambda_j^{(n)}) e_2(\lambda_i^{(n)} - \lambda_j^{(n)}) e_2(\lambda_i^{(n)} + \lambda_j^{(n)}) \\
&\quad \times \prod_{j=1}^{M^{(n-1)}} e_{-1}(\lambda_i^{(n)} - \lambda_j^{(n-1)}) e_{-1}(\lambda_i^{(n)} + \lambda_j^{(n-1)}).
\end{aligned} \tag{4}$$

Note that in this case the Bethe ansatz equations are similar to the ones of the open $\mathfrak{osp}(1|2n)$ spin chain (see also [6], [8], [9]).

- $\mathfrak{sl}(2n)$

$$\begin{aligned}
e_1^L(\lambda_i^{(1)}) &= - \prod_{j=1}^{M^{(1)}} e_2(\lambda_i^{(1)} - \lambda_j^{(1)}) e_2(\lambda_i^{(1)} + \lambda_j^{(1)}) \prod_{j=1}^{M^{(2)}} e_{-1}(\lambda_i^{(1)} - \lambda_j^{(2)}) e_{-1}(\lambda_i^{(1)} + \lambda_j^{(2)}), \\
1 &= - \prod_{j=1}^{M^{(\ell)}} e_2(\lambda_i^{(\ell)} - \lambda_j^{(\ell)}) e_2(\lambda_i^{(\ell)} + \lambda_j^{(\ell)}) \prod_{\tau=\pm 1} \prod_{j=1}^{M^{(\ell+\tau)}} e_{-1}(\lambda_i^{(\ell)} - \lambda_j^{(\ell+\tau)}) e_{-1}(\lambda_i^{(\ell)} + \lambda_j^{(\ell+\tau)}) \\
&\ell = 2, \dots, n-1, \\
e_{-1}(\lambda_i^{(n)}) &= - \prod_{j=1}^{M^{(n)}} e_2(\lambda_i^{(n)} - \lambda_j^{(n)}) e_2(\lambda_i^{(n)} + \lambda_j^{(n)}) \\
&\quad \times \prod_{j=1}^{M^{(n-1)}} e_{-1}^2(\lambda_i^{(n)} - \lambda_j^{(n-1)}) e_{-1}^2(\lambda_i^{(n)} + \lambda_j^{(n-1)}).
\end{aligned} \tag{5}$$

As opposed to the $\mathfrak{sl}(2n+1)$ case the Bethe ansatz equations above do not reduce to any of the known forms of BAE, which makes the whole study even more intriguing.

Note that the number $M^{(l)}$ are associated to the eigenvalues of the diagonal generators S_l of the underlying algebra $\mathfrak{so}(n)$ (see [6, 8] for a detailed discussion on the underlying symmetry of the models), i.e.

$$S_1 = \frac{1}{2}M^{(0)} - M^{(1)}, \quad S_l = M^{(l-1)} - M^{(l)}, \quad S_l = \frac{1}{2}(E_{ll} - E_{\bar{l}\bar{l}}), \quad l \leq l \leq \frac{\mathcal{N}-1}{2} \tag{6}$$

E_{ll} the diagonal generators of $\mathfrak{sl}(\mathcal{N})$, and $\bar{l} = \mathcal{N} - l + 1$ the conjugate index.

It is also worth recalling that the corresponding numbers in the usual $\mathfrak{sl}(\mathcal{N})$ case are given by:

$$E_{ll} = M^{(l-1)} - M^{(l)}, \quad M^{(0)} = 2L, \quad M^{(\mathcal{N})} = 0, \quad l \in \{1, 2, \dots, \mathcal{N}\} \quad (7)$$

By imposing $M^{(l)} = M^{(\mathcal{N}-1)}$ and considering the differences $E_{ll} - E_{\bar{l}\bar{l}}$ we end up to (6) in accordance to the folding of $\mathfrak{sl}(\mathcal{N})$ leading to the $\mathfrak{so}(n)$ algebra [6, 8].

The ground state of the model consists of n filled Dirac seas, unlike the Yangian case, where the ground state consists of $2n + 1$ or $2n$ filled seas respectively. The number of seas is halved here due to the underlying algebraic folding (see also [8]). As usual, an excitation corresponds to a hole in the Dirac sea. We perform our computations in the thermodynamic limit of the BAE, which is obtained according to the thermodynamic rule (for more details the interested reader is referred to e.g. [11, 12] or [5] in a more relevant context)

$$\frac{1}{L} \sum_{j=1}^{M^{(\ell)}} f(\lambda_j^{(\ell)}) \rightarrow \int_0^\infty d\mu \sigma_\ell(\mu) f(\mu) - \frac{1}{L} \sum_{j=1}^{\nu^{(\ell)}} f(\tilde{\lambda}_j^{(\ell)}) - \frac{1}{2L} f(0), \quad (8)$$

with $\nu^{(\ell)}$ holes of rapidities $\tilde{\lambda}_j^{(\ell)}$ in the ℓ^{th} Dirac sea σ_ℓ is the density in the ℓ^{th} sea. The last term is the halved contribution at 0^+ due to the boundaries. In the thermodynamic limit the BAE take the compact form

$$\begin{aligned} \hat{\mathcal{K}}(\omega) \hat{\sigma}(\omega) &= \hat{\mathcal{A}}(\omega) + \frac{1}{L} \left(\hat{\mathcal{F}}^{(1)}(\omega) + \mathcal{F}^{(2)}(\omega) \right) \\ \Rightarrow \hat{\sigma}(\omega) &= \hat{\sigma}^{(0)}(\omega) + \frac{1}{L} \left(r^{(1)}(\omega) + r^{(2)}(\omega) \right), \end{aligned} \quad (9)$$

where we have defined the n -dimensional column vectors $\hat{\sigma}$, $\hat{\mathcal{A}}$ and $\hat{\mathcal{F}}^{(i)}$ with elements

$$\hat{\sigma}_i(\omega) \quad \hat{\mathcal{A}}_i(\omega) = \hat{a}_1(\omega) \delta_{i1}, \quad i \in \{1, 2, \dots, n\}, \quad (10)$$

also we define

$$a_n(\lambda) = \frac{i}{2\pi} \frac{d}{d\lambda} (\ln e_n(\lambda)), \quad \hat{a}_n(\omega) = e^{-\frac{n|\omega|}{2}} \quad (11)$$

and

- $\mathfrak{sl}(2n + 1)$

$$\begin{aligned} \hat{\mathcal{F}}_i^{(1)}(\omega) &= \hat{a}_1(\omega) \delta_{i1} - 2\hat{a}_1(\omega) + \hat{a}_2(\omega) - \hat{a}_{\frac{1}{2}}(\omega) \delta_{in} \\ \hat{\mathcal{F}}_i^{(2)}(\omega) &= 2 \left(\hat{a}_2(\omega) - \hat{a}_1(\omega) \delta_{in} \right) \sum_{j=1}^{\nu^{(i)}} \cos(\omega \tilde{\lambda}_j^{(i)}) - 2\hat{a}_1(\omega) \sum_{j=1}^{\nu^{(k)}} \cos(\omega \tilde{\lambda}_j^{(k)}) (\delta_{k,i+1} + \delta_{k,i-1}). \end{aligned} \quad (12)$$

The kernel $\hat{\mathcal{K}}(\omega)$ is the $n \times n$ matrix

$$\hat{\mathcal{K}}_{ij}(\omega) = (1 - \hat{a}_1(\omega) \delta_{in} + \hat{a}_2(\omega)) \delta_{ij} - \hat{a}_1(\omega) (\delta_{i,j+1} + \delta_{i,j-1}), \quad i, j \in \{1, \dots, n\}, \quad (13)$$

and the entries of its inverse are given by

$$\hat{\mathcal{R}}_{ij}(\omega) = e^{\frac{\omega}{2}} \frac{\sinh\left(\min(i, j)\frac{\omega}{2}\right) \cosh\left(n + \frac{1}{2} - \max(i, j)\right)\frac{\omega}{2}}{\cosh\left(n + \frac{1}{2}\right)\frac{\omega}{2} \sinh\frac{\omega}{2}}. \quad (14)$$

- $\mathfrak{sl}(2n)$

In this case as well the ground state consists of n filled Dirac seas. The thermodynamic limit of the BAE leads to the densities of Bethe roots (see expressions (12)) with $\hat{\mathcal{F}}^{(i)}$ now defined accordingly as

$$\begin{aligned} \hat{\mathcal{F}}_i^{(1)}(\omega) &= \hat{a}_2(\omega) - (2 - \delta_{i1} + \delta_{in}) \hat{a}_1(\omega) \\ \hat{\mathcal{F}}_i^{(2)}(\omega) &= -2\hat{a}_1(\omega) \sum_{j=1}^{\nu^{(k)}} \cos(\omega \tilde{\lambda}_j^{(k)}) (\delta_{k,i+1} + \delta_{k,i-1}(1 + \delta_{in})) + 2\hat{a}_2(\omega) \sum_{j=1}^{\nu^{(i)}} \cos(\omega \tilde{\lambda}_j^{(i)}). \end{aligned} \quad (15)$$

The kernel $\hat{\mathcal{K}}$ is given by the $n \times n$ matrix with elements

$$\mathcal{K}_{ij}(\omega) = (1 + \hat{a}_2(\omega))\delta_{ij} - \hat{a}_1(\omega)(\delta_{i,j+1} + \delta_{i,j-1}) - \hat{a}_1(\omega)\delta_{in}\delta_{j,n-1}, \quad (16)$$

and its inverse by

$$\hat{\mathcal{R}}_{ij}(\omega) = e^{\frac{\omega}{2}} \frac{\sinh\left(\min(i, j)\frac{\omega}{2}\right) \cosh\left((n - \max(i, j))\frac{\omega}{2}\right)}{(1 + \delta_{jn}) \cosh\left(\frac{n\omega}{2}\right) \sinh\left(\frac{\omega}{2}\right)}. \quad (17)$$

Having set all the necessary ingredients we are now in a position to proceed with the thermodynamic computations of the energy of particle-like excitations, as well as their scattering amplitudes. Later in the text local defects will be introduced and the scattering between the excitation and the defects will be discussed in detail.

2.1 The energy

In this section the energy of particle-like excitations (holes in the ℓ^{th} sea) is derived. From the eigenvalues of the transfer matrix, the energy of the system may be derived taking the first derivative of the transfer matrix eigenvalues with respect to the spectral parameter (see also e.g. [5] for a more detailed computation).

As a validity check on the form of the ground state and excitations, together with the quantization condition derived in [5] (see also next section), we compute the energy of a single hole in the j^{th} sea and compare with the resulting expression for the density $\hat{\sigma}_j^{(0)}$. These quantities should be the same so that the quantization condition may be appropriately employed. It is worth noting that the a single excitations here is associated to representations of the underlying exact symmetry which is $\mathfrak{so}(n)$ it is indeed a “folded” algebra (a folding at the level of Dynkin diagram occurs) compared to $\mathfrak{sl}(2n)$, $\mathfrak{sl}(2n+1)$ as extensively discussed

in [6, 8], see also as similar discussion and examples on the quantum numbers of excitations in [5].

The energy derived in the thermodynamic is given by the following expression (see also [5]):

$$\epsilon = - \int_0^\infty d\mu a_1(\mu) \sigma_1(\mu) + \frac{1}{L} a_1(\tilde{\lambda}_1) - \frac{1}{2L} a_1(0). \quad (18)$$

The energy of a particle-like excitation (hole) in the j^{th} sea in particular is then given as (its Fourier transform)

$$\hat{\epsilon}^{(j)}(\omega) = -\hat{a}_1(\omega) \hat{r}_j^{(2)}(\omega) + \hat{a}_1(\omega) \delta_{j1}, \quad (19)$$

whereas the density $\hat{\sigma}_j^{(0)}$ is given by

$$\hat{\sigma}_j^{(0)} = \hat{\mathcal{R}}_{j1}(\omega) \hat{a}_1(\omega). \quad (20)$$

We define the $\hat{r}_j^{(2)}(\omega)$ quantities as follows:

- $\mathfrak{sl}(2n+1)$

$$\begin{aligned} \hat{r}_1^{(2)}(\omega) &= \hat{R}_{11}(\omega) \hat{a}_2(\omega) - \hat{a}_1(\omega) \hat{R}_{12}(\omega), \quad \text{hole in the } 1^{st} \text{ sea} \\ \hat{r}_j^{(2)}(\omega) &= \hat{R}_{1j}(\omega) \hat{a}_2(\omega) - \hat{a}_1(\omega) (\hat{R}_{1j+1}(\omega) + \hat{R}_{1j-1}(\omega)), \quad \text{hole in the } j^{th} \text{ sea } (j \neq 1, n) \\ \hat{r}_n^{(2)}(\omega) &= \hat{R}_{1n}(\omega) (\hat{a}_2(\omega) - \hat{a}_1(\omega)) - \hat{a}_1(\omega) \hat{R}_{1n-1}(\omega), \quad \text{hole in the } n^{th} \text{ sea}. \end{aligned} \quad (21)$$

It turns out that the energy of a hole in the j^{th} sea is given as:

$$\hat{\epsilon}^{(j)}(\omega) = \frac{\cosh(n + \frac{1}{2} - j) \frac{\omega}{2}}{\cosh(n + \frac{1}{2}) \frac{\omega}{2}}, \quad j \in \{1, 2, \dots, n\} \quad (22)$$

whereas the density $\hat{\sigma}_j^{(0)}$ is computed to be:

$$\hat{\sigma}_j^{(0)}(\omega) = \hat{\mathcal{R}}_{j1}^j(\omega) \hat{a}_1(\omega) = \frac{\cosh(n + \frac{1}{2} - j) \frac{\omega}{2}}{\cosh(n + \frac{1}{2}) \frac{\omega}{2}}, \quad j \in \{1, \dots, n\}. \quad (23)$$

- $\mathfrak{sl}(2n)$

The expression of the energy of a hole in the j^{th} sea is given by the same expressions as in the odd case with the exception of the holes in the n and $n-1$ seas:

$$\begin{aligned} \hat{r}_{n-1}^{(2)}(\omega) &= \hat{R}_{1n-1}(\omega) \hat{a}_2(\omega) - \hat{a}_1(\omega) (2\hat{R}_{1n}(\omega) + \hat{R}_{1n-2}(\omega)), \quad \text{hole in the } (n-1)^{th} \text{ sea } (j \neq 1, n) \\ \hat{r}_n^{(2)}(\omega) &= \hat{R}_{1n}(\omega) \hat{a}_2(\omega) - \hat{a}_1(\omega) \hat{R}_{1n-1}(\omega), \quad \text{hole in the } n^{th} \text{ sea}. \end{aligned} \quad (24)$$

In this case the energy of a hole in the j^{th} sea is given as:

$$\begin{aligned} \hat{\epsilon}^{(j)}(\omega) &= \frac{\cosh(n-j) \frac{\omega}{2}}{\cosh \frac{n\omega}{2}}, \quad j \in \{1, 2, \dots, n-1\}, \\ \hat{\epsilon}^{(n)}(\omega) &= \frac{1}{2 \cosh \frac{n\omega}{2}}, \end{aligned} \quad (25)$$

and the density $\hat{\sigma}_j^{(0)}$ is given by:

$$\hat{\sigma}_j^{(0)}(\omega) = \hat{\mathcal{R}}_{j1}(\omega) \hat{a}_1(\omega) = \frac{\cosh(n-j)\frac{\omega}{2}}{\cosh \frac{n\omega}{2}}, \quad j \in \{1, \dots, n-1\}. \quad (26)$$

Having verified the fact that for each particle-like excitation the equation: $\hat{\sigma}_j^{(0)}(\omega) = \hat{e}^{(j)}(\omega)$, is valid and compatible with the quantization condition [5], we now derive the associated bulk and boundary scattering amplitudes.

3 Scattering amplitudes

The key element in this context is now the generalized quantization condition for the twisted Yangian introduced in [5]. We shall consider here the scattering of particle-like excitations in the first sea. Recall that the quantization condition (see also [10], [11], [12]) for a state with two holes in the ℓ^{th} sea reads as [5]

$$\left(e^{i\mathcal{P}^{(\ell)}L} \mathbb{S}(\tilde{\lambda}_1^{(\ell)}, \tilde{\lambda}_2^{(\ell)}) - 1 \right) |\tilde{\lambda}_1^{(\ell)} \tilde{\lambda}_2^{(\ell)}\rangle = 0, \quad (27)$$

where $\mathcal{P}^{(\ell)}$ is the momentum of the hole in the ℓ^{th} sea, and the global scattering matrix \mathbb{S} is given by [5]

$$\mathbb{S}(\lambda_1, \lambda_2) = \mathcal{K}^+(\lambda_1) \mathcal{S}(\lambda_1 - \lambda_2) \mathcal{K}^-(\lambda_2) \mathcal{S}(\lambda_1 + \lambda_2), \quad (28)$$

where $\mathcal{S}(\lambda) = S(\lambda) \bar{S}(\lambda)$ and S (\bar{S}) corresponds to the soliton–soliton (soliton–anti-soliton) scattering amplitude of the Yangian $\mathfrak{sl}(\mathcal{N})$. This bulk factorization will be explicitly shown below. \mathcal{K}^\pm are the physical boundary scattering matrices associated with the left/right boundaries of the system. We have considered here for simplicity the left/right boundary matrices $\mathbb{K}^\pm \propto \mathbb{I}$. Note that here we obtain the eigenvalue associated to the hole-hole and hole-boundary interactions. Note that more eigenvalues can be identified using complex “string” type solutions of the BAE, however such an analysis is beyond the intended scope of the present article, given especially the algebraic arguments leading to the factorized structure of the scattering matrix (see also [5]).

Indeed the validity of the factorization (and the quantization condition for that matter) at the matrix level as well as the form of the S and \bar{S} matrices are confirmed by the underlying algebra as well as the quantization condition (for similar algebraic arguments see [15], [5]). It is in any case well established that the Bethe ansatz formulation serves as a “renormalization” process, thus S , \bar{S} and \mathcal{K} matrices are basically “renormalized” (physical) quantities as opposed to the “bare” R , \bar{R} and \mathbb{K}^\pm . Bethe ansatz provides essentially the overall physical factors S_0 , \bar{S}_0 and K_0^\pm , and this is exactly what we perform in what follows.

Using the dispersion relation

$$\epsilon^{(\ell)}(\lambda) = \frac{1}{2\pi} \frac{d}{d\lambda} \mathcal{P}^{(\ell)}(\lambda), \quad (29)$$

and the fact that $L \int_0^{\tilde{\lambda}_i^{(\ell)}} d\lambda \sigma(\lambda) \in \mathbb{Z}$, we conclude that the scattering matrix phase, ($\mathbb{S} = \exp(i\Phi)$), is given by

$$i\Phi = - \int_{-\infty}^{\infty} \frac{d\omega}{\omega} e^{-i\omega\tilde{\lambda}_1} (\hat{\sigma}_1(\omega) - \hat{\epsilon}^{(1)}(\omega)). \quad (30)$$

Introducing two excitations (holes) in the first sea gives

$$i\Phi = - \int_{-\infty}^{\infty} \frac{d\omega}{\omega} e^{-i\omega\tilde{\lambda}_1} \sum_{j=1}^n \hat{\mathcal{R}}_{1j}(\omega) \hat{\mathcal{F}}_j(\omega). \quad (31)$$

Recalling the expression for $\hat{\mathcal{F}}^{(i)}$, and keeping in mind that we consider two holes in the first sea, we need the following quantities in the summation above:

$$\begin{aligned} \hat{\mathcal{F}}_1^{(1)}(\omega) &= -\hat{a}_1(\omega) + \hat{a}_2(\omega), & \hat{\mathcal{F}}_1^{(2)}(\omega) &= \hat{a}_2(\omega) \sum_{j=1}^2 (e^{i\omega\tilde{\lambda}_j^{(1)}} + e^{-i\omega\tilde{\lambda}_j^{(1)}}) \\ \hat{\mathcal{F}}_2^{(1)}(\omega) &= -2\hat{a}_1(\omega) + \hat{a}_2(\omega), & \hat{\mathcal{F}}_2^{(2)}(\omega) &= -\hat{a}_1(\omega) \sum_{j=1}^2 (e^{i\omega\tilde{\lambda}_j^{(1)}} + e^{-i\omega\tilde{\lambda}_j^{(1)}}) \\ \hat{\mathcal{F}}_k^{(1)}(\omega) &= -2\hat{a}_1(\omega) + \hat{a}_2(\omega), & \hat{\mathcal{F}}_k^{(2)}(\omega) &= 0, \quad k = 3, \dots, n-1, \\ \hat{\mathcal{F}}_n^{(1)}(\omega) &= \begin{cases} -2\hat{a}_1(\omega) + \hat{a}_2(\omega) - \hat{a}_{\frac{1}{2}}(\omega), & \mathfrak{sl}(2n+1) \\ -3\hat{a}_1(\omega) + \hat{a}_2(\omega), & \mathfrak{sl}(2n) \end{cases}, & \hat{\mathcal{F}}_n^{(2)}(\omega) &= 0. \end{aligned} \quad (32)$$

We identify then the bulk and boundary scattering amplitudes as

$$\begin{aligned} \mathcal{S}_0(\lambda) &= \exp \left\{ - \int_{-\infty}^{\infty} \frac{d\omega}{\omega} e^{-i\omega\lambda} \mathcal{B}_1(\omega) \right\} \\ K_0^+(\lambda) K_0^-(\lambda) &= \exp \left\{ - \int_{-\infty}^{\infty} \frac{d\omega}{\omega} \left(e^{-i\omega\tilde{\lambda}^{(1)}} \mathcal{B}_2(\omega) + e^{-2i\omega\lambda} \mathcal{B}_1(\omega) \right) \right\}, \end{aligned} \quad (33)$$

where the terms \mathcal{B}_i contain the collected contributions from $\mathcal{F}^{(j)}$ and \mathcal{R} and read

$$\begin{aligned} \mathcal{B}_1(\omega) &= \hat{a}_2(\omega) \hat{\mathcal{R}}_{11}(\omega) - \hat{a}_1(\omega) \hat{\mathcal{R}}_{12}(\omega) \\ \mathcal{B}_2(\omega) &= \sum_{j=1}^n (\hat{a}_2(\omega) - 2\hat{a}_1(\omega) + \hat{a}_1(\omega)\delta_{i1} - \hat{a}_{\frac{1}{2}}(\omega)\delta_{in}) \hat{\mathcal{R}}_{1i}(\omega). \end{aligned} \quad (34)$$

It is worth noting that explicit results on the generic boundary scattering amplitude for the twisted Yangian are presented here for the first time (33), (34), although similar computations regarding the $\mathfrak{sl}(3)$ case were performed in [5]. In the following we use the expressions on bulk scattering to explicitly show its factorization.

As in the $\mathfrak{sl}(3)$ case [5], it can be shown that the bulk scattering factorizes to a product of a soliton–soliton times a soliton–anti-soliton scattering amplitude of the usual Yangian

model. Consider two holes in the first sea, for the twisted Yangian case. The bulk scattering phase in the $\mathfrak{sl}(\mathcal{N})$ ($\mathcal{N} = 2n$ or $2n + 1$) twisted Yangian is given by

$$\mathcal{B}_S(\omega) = \hat{\mathcal{R}}_{11}(\omega) \hat{a}_2(\omega) - \hat{\mathcal{R}}_{12}(\omega) \hat{a}_1(\omega) = \frac{1 - e^\omega + e^{-\frac{\mathcal{N}\omega}{2} + \omega} - e^{-\frac{\mathcal{N}\omega}{2}}}{2 \sinh \frac{\mathcal{N}\omega}{2}}. \quad (35)$$

In the usual $\mathfrak{sl}(\mathcal{N})$ case the soliton–soliton bulk scattering amplitude is [13]

$$\mathcal{B}_S(\omega) = \frac{e^{-\omega(\frac{\mathcal{N}}{2}-1)} - e^{-\frac{\mathcal{N}\omega}{2}}}{2 \sinh(\frac{\mathcal{N}\omega}{2})}, \quad (36)$$

while for the soliton–anti-soliton we have [13]

$$\mathcal{B}_{\bar{S}}(\omega) = \frac{1 - e^\omega}{2 \sinh(\frac{\mathcal{N}\omega}{2})}. \quad (37)$$

The factorization of the scattering phase is then immediately observed

$$\mathcal{B}_S(\omega) = \mathcal{B}_S(\omega) + \mathcal{B}_{\bar{S}}(\omega), \quad (38)$$

leading to the factorization of the bulk part of the scattering

$$\mathcal{S}_0(\lambda) = S_0(\lambda) \times \bar{S}_0(\lambda), \quad (39)$$

where recall that S_0 , \bar{S}_0 are the soliton–soliton and soliton–anti-soliton scattering amplitudes in the Yangian $\mathfrak{sl}(\mathcal{N})$ case, expressed as:

$$\mathcal{X}(\lambda) = \exp \left[- \int_{-\infty}^{\infty} \frac{d\omega}{\omega} e^{-i\omega\lambda} \mathcal{B}_X(\omega) \right], \quad \mathcal{X} \in \{S, \bar{S}, \mathcal{S}\}. \quad (40)$$

We have shown that the factorization of the bulk scattering, such as was first observed in the $\mathfrak{sl}(3)$ twisted Yangian case [5], is valid in the generic case as well. With this we conclude our discussion on the bulk and boundary scattering in the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian.

4 Implementing defects

We shall focus henceforth on the study of defect-transmission amplitudes in the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian. As is well known, transmission matrices physically describe the interaction between the particle-like excitation of the model and the defect. It will be instructive in this section to introduce some basic notions on the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian in the presence of defects. The defect matrix used in this case takes of the generic form:

$$\mathbb{L}(\lambda) = \lambda + i\mathbb{P}, \quad \mathbb{P} = \sum_{i,j=1}^{\mathcal{N}} e_{ij} P_{ij}, \quad (41)$$

e_{ij} are $\mathcal{N} \times \mathcal{N}$ matrices such that $(e_{ij})_{kl} = \delta_{ik} \delta_{jl}$, and P_{ij} are the $\mathfrak{gl}(\mathcal{N})$ algebra generators

$$[P_{ij}, P_{kl}] = \delta_{il} P_{kj} - \delta_{kj} P_{il}. \quad (42)$$

The Yangian R -matrix is given by [14]

$$R(\lambda) = \lambda + i\mathcal{P}, \quad \mathcal{P} = \sum_{i,j=1}^{\mathcal{N}} e_{ij} \otimes e_{ji}, \quad (43)$$

\mathcal{P} being the $\mathfrak{gl}(\mathcal{N})$ permutation operator. The R -matrix above is associated with the fundamental representation of $\mathfrak{gl}(\mathcal{N})$.

A generic finite-irreducible representation of the $\mathfrak{gl}(\mathcal{N})$ algebra is associated with \mathcal{N} integers $(\alpha_1, \alpha_2, \dots, \alpha_{\mathcal{N}})$, $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_{\mathcal{N}}$. Here we deal with representations that possess highest weight state such that:

$$\begin{aligned} P_{kl} |\omega\rangle_n &= 0, & k < l \\ P_{kk} |\omega\rangle_n &= \alpha_k |\omega\rangle_n \\ e_{kl} |\omega\rangle_j &= 0, & k > l, \quad j \neq n \\ e_{kk} |\omega\rangle_j &= \omega_j |\omega\rangle_j. \end{aligned} \quad (44)$$

The global reference state is then (note that we consider henceforth for convenience a chain with $L + 1$ sites to incorporate the defect)

$$|\Omega\rangle = \bigotimes_{j=1}^{L+1} |\omega\rangle_j. \quad (45)$$

We shall also need the conjugate \mathbb{L} matrix derived as

$$\bar{\mathbb{L}}_{12}(\lambda) = V_1 \mathbb{L}_{12}^{t_1}(-\lambda - \frac{i\mathcal{N}}{2}) V_1, \quad V = \text{antidiag}(1, \dots, 1) \quad (46)$$

where t_1 denotes transposition on space 1, and hence

$$\bar{\mathbb{L}}(\lambda) = \lambda + \frac{i\mathcal{N}}{2} - i\bar{\mathbb{P}}, \quad \bar{\mathbb{P}}_{12} = V_1 \mathbb{P}_{12}^{t_1} V_1. \quad (47)$$

Note that physically the \mathbb{L} matrix corresponds to the defect whereas the $\bar{\mathbb{L}}$ matrix correspond to what we figuratively call the anti-defect.

The transfer matrix of the twisted Yangian in the presence of defects reads as:

$$\begin{aligned} t(\lambda) &= \text{tr}_0 \left(\mathbb{K}_0^+(\lambda) T_0(\lambda) \mathbb{K}_0^-(\lambda) V_0 T_0^{t_0}(-\lambda - \frac{i\mathcal{N}}{2}) V_0 \right) \\ T_0(\lambda) &= R_{0L+1}(\lambda) R_{0L}(\lambda) \dots \mathbb{L}_{0n}(\lambda - \Theta) \dots R_{01}(\lambda) \end{aligned} \quad (48)$$

R is the $\mathfrak{gl}(\mathcal{N})$ Yangian matrix (43), Θ is the rapidity associated to the defect. Recall that we have considered for simplicity here, and throughout the text that $\mathbb{K}^{\pm} \propto \mathbb{I}$, and it is clear

that the defect is inserted in the n^{th} site of the chain. It is also worth mentioning at this point that in order to obtain local Hamiltonians one needs to consider the alternating spin chain (alternate R and \bar{R} matrices) (see e.g. [6], [8]), the spectrum as well as the BAE are not modified in this case. The corresponding Hamiltonians in this case have been explicitly derived in [6], [8] and contain terms that describe interactions up to four neighbours. Here of course we have to take into consideration the defect contributions which give rise to relevant terms that describe interactions up to six neighbours. Nevertheless, it is important to note that one can still obtain a local Hamiltonian via the usual process of taking the first derivative of the logarithm of the transfer matrix.

Assuming the existence of highest weight states which is the case here, the formulation of the spectrum and Bethe ansatz equations follows the same logic described in [6], [8], and in the case where defects are present the BAE are modified accordingly as:

- $\mathfrak{sl}(2n+1)$

$$\begin{aligned}
& X_1^+(\lambda_i^{(1)} - \Theta) X_1^+(\lambda_i^{(1)} + \Theta) e_1^L(\lambda_i^{(1)}) = \\
& \quad - \prod_{j=1}^{M^{(1)}} e_2(\lambda_i^{(1)} - \lambda_j^{(1)}) e_2(\lambda_i^{(1)} + \lambda_j^{(1)}) \prod_{j=1}^{M^{(2)}} e_{-1}(\lambda_i^{(1)} - \lambda_j^{(2)}) e_{-1}(\lambda_i^{(1)} + \lambda_j^{(2)}), \\
& X_\ell^+(\lambda_i^{(l)} - \Theta) X_\ell^+(\lambda_i^{(l)} + \Theta) = \\
& \quad - \prod_{j=1}^{M^{(\ell)}} e_2(\lambda_i^{(\ell)} - \lambda_j^{(\ell)}) e_2(\lambda_i^{(\ell)} + \lambda_j^{(\ell)}) \prod_{\tau=\pm 1} \prod_{j=1}^{M^{(\ell+\tau)}} e_{-1}(\lambda_i^{(\ell)} - \lambda_j^{(\ell+\tau)}) e_{-1}(\lambda_i^{(\ell)} + \lambda_j^{(\ell+\tau)}) \\
& \quad \text{for } \ell = 2, \dots, n-1, \\
& X_n^+(\lambda_i^{(n)} - \Theta) X_n^+(\lambda_i^{(n)} + \Theta) e_{-\frac{1}{2}}(\lambda_i^{(n)}) = - \prod_{j=1}^{M^{(n-1)}} e_{-1}(\lambda_i^{(n)} - \lambda_j^{(n-1)}) e_{-1}(\lambda_i^{(n)} + \lambda_j^{(n-1)}) \\
& \quad \times - \prod_{j=1}^{M^{(n)}} e_{-1}(\lambda_i^{(n)} - \lambda_j^{(n)}) e_{-1}(\lambda_i^{(n)} + \lambda_j^{(n)}) e_2(\lambda_i^{(n)} - \lambda_j^{(n)}) e_2(\lambda_i^{(n)} + \lambda_j^{(n)}) \quad (49)
\end{aligned}$$

- $\mathfrak{sl}(2n)$

$$\begin{aligned}
& X_1^+(\lambda_i^{(1)} - \Theta) X_1^-(\lambda_i^{(1)} + \Theta) e_1^L(\lambda_i^{(1)}) = \\
& \quad - \prod_{j=1}^{M^{(1)}} e_2(\lambda_i^{(1)} - \lambda_j^{(1)}) e_2(\lambda_i^{(1)} + \lambda_j^{(1)}) \prod_{j=1}^{M^{(2)}} e_{-1}(\lambda_i^{(1)} - \lambda_j^{(2)}) e_{-1}(\lambda_i^{(1)} + \lambda_j^{(2)}), \\
& X_\ell^+(\lambda_i^{(l)} - \Theta) X_\ell^-(\lambda_i^{(l)} + \Theta) =
\end{aligned}$$

$$\begin{aligned}
& - \prod_{j=1}^{M^{(\ell)}} e_2(\lambda_i^{(\ell)} - \lambda_j^{(\ell)}) e_2(\lambda_i^{(\ell)} + \lambda_j^{(\ell)}) \prod_{\tau=\pm 1} \prod_{j=1}^{M^{(\ell+\tau)}} e_{-1}(\lambda_i^{(\ell)} - \lambda_j^{(\ell+\tau)}) e_{-1}(\lambda_i^{(\ell)} + \lambda_j^{(\ell+\tau)}) \\
& \text{for } \ell = 2, \dots, n-1, \\
& X_n^+(\lambda_i^{(n)} - \Theta) X_n^-(\lambda_i^{(n)} + \Theta) e_{-1}(\lambda_i^{(n)}) = - \prod_{j=1}^{M^{(n)}} e_2(\lambda_i^{(n)} - \lambda_j^{(n)}) e_2(\lambda_i^{(n)} + \lambda_j^{(n)}) \\
& \times \prod_{j=1}^{M^{(n-1)}} e_{-1}^2(\lambda_i^{(n)} - \lambda_j^{(n-1)}) e_{-1}^2(\lambda_i^{(n)} + \lambda_j^{(n-1)}), \tag{50}
\end{aligned}$$

where X_k^\pm are the defect contributions (the action of \mathbb{L}_{kk} and $\hat{\mathbb{L}}_{kk}$ on the local highest weight state (44)) and are defined as:

$$X_k^+(\lambda) = \frac{\lambda + i\alpha_k - \frac{ik}{2}}{\lambda + i\alpha_{k+1} - \frac{ik}{2}}, \quad X_k^-(\lambda) = \frac{\lambda - i\alpha_{\mathcal{N}-k+1} + \frac{i(\mathcal{N}-k)}{2}}{\lambda - i\alpha_{\mathcal{N}-k} + \frac{i(\mathcal{N}-k)}{2}}. \tag{51}$$

Having derived the associated BAEs we now formulate a suitable quantization condition for the model in the presence of defects. In order to determine the relevant transmission matrix it suffices to consider a state with one hole in the first sea. Before we discuss the quantization condition in this case, let us first introduce some notation and define the transmission amplitudes in $\mathfrak{gl}(\mathcal{N})$ [15] as

$$\begin{aligned}
T(\lambda - \Theta) &: \text{soliton--defect scattering} \\
\bar{T}(\lambda - \Theta) &: \text{soliton--anti-defect scattering} \\
T^*(\lambda + \Theta) &: \text{anti-soliton--defect scattering} \\
\bar{T}^*(\lambda + \Theta) &: \text{anti-soliton--anti-defect scattering}
\end{aligned} \tag{52}$$

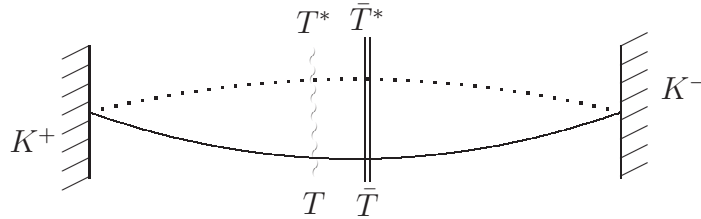
The quantization condition for such a state reads as

$$\left(e^{i\mathcal{P}^{(l)}} \mathbb{S}(\tilde{\lambda}^{(l)}, \Theta) - 1 \right) |\tilde{\lambda}^{(l)}, \Theta\rangle = 0, \tag{53}$$

where the global scattering amplitude is given by

$$\mathbb{S}(\lambda, \Theta) = \mathcal{K}^+(\lambda) T(\lambda - \Theta) \bar{T}(\lambda - \Theta) \mathcal{K}^-(\lambda) \bar{T}^*(\lambda + \Theta) T^*(\lambda + \Theta) \tag{54}$$

The latter can be pictorially represented as:



The factorization of transmission amplitudes will be transparent in the following. Note that the boundary scattering remains unaffected and is given in the previous section. The

phase associated to the global defect–particle interaction in the twisted Yangian is provided by

$$\mathcal{B}_{\mathbb{T}}(\omega) = \sum_{k=1}^n \hat{\mathcal{R}}_{1k}(\omega) \left(\hat{Y}_k^+(\omega) e^{i\omega\Theta} + \hat{Y}_k^-(\omega) e^{-i\omega\Theta} \right), \quad (55)$$

where we define

$$Y_k^{\pm}(\lambda) = \frac{i}{2\pi} \frac{dX_k^{\pm}(\lambda)}{d\lambda}. \quad (56)$$

More precisely,

$$\begin{aligned} Y_k^+(\lambda) &= a\left(\alpha_k - \frac{k}{2}, \alpha_{k+1} - \frac{k}{2}; \lambda\right) \\ Y_k^-(\lambda) &= a\left(-\alpha_{\mathcal{N}-k+1} + \frac{\mathcal{N}-k}{2}, -\alpha_{\mathcal{N}-k} + \frac{\mathcal{N}-k}{2}; \lambda\right) \end{aligned} \quad (57)$$

and we define

$$a(x, y; \lambda) = \frac{i}{2\pi} \left(\frac{1}{\lambda + ix} - \frac{1}{\lambda + iy} \right). \quad (58)$$

The Fourier transforms of the latter expressions are given below. We distinguish three cases according to the values of x , y , and we end up with the following Fourier transforms in the isotropic case (see also [15]):

$$\begin{aligned} \hat{a}(x, y; \omega) &= e^{\omega x} \quad \omega < 0, \quad \hat{a}(x, y; \omega) = e^{\omega y} \quad \omega > 0, \quad x > 0, \quad y < 0 \\ \hat{a}(x, y; \omega) &= e^{\omega y} - e^{\omega x} \quad \omega > 0, \quad \hat{a}(x, y; \omega) = 0 \quad \omega < 0, \quad x, y < 0 \\ \hat{a}(x, y; \omega) &= e^{\omega x} - e^{\omega y} \quad \omega < 0, \quad \hat{a}(x, y; \omega) = 0 \quad \omega > 0, \quad x, y > 0. \end{aligned} \quad (59)$$

It is clear that in the special case $x = -y = \frac{n}{2}$ the expressions above reduce to the familiar Fourier transforms.

The important observation at this point here is that

$$\hat{\mathcal{R}}_{1k}(\omega) = \hat{R}_{1k}(\omega) + \hat{R}_{\mathcal{N}-1k}(\omega), \quad (60)$$

where \hat{R} is the Fourier transform of the inverse kernel in the $\mathfrak{gl}(\mathcal{N})$ Yangian [13]

$$\hat{R}_{ij}(\omega) = e^{\frac{|\omega|}{2}} \frac{\sinh(\min(i, j) \frac{\omega}{2}) \sinh(\mathcal{N} - \max(i, j) \frac{\omega}{2})}{\sinh \frac{\omega}{2} \sinh(\frac{\mathcal{N}\omega}{2})}. \quad (61)$$

The latter identity (60) naturally leads to the factorization of the transmission amplitudes as depicted diagrammatically in the quantization condition figure. Indeed, as discussed in [15], the phases associated to transmission amplitudes are derived as

$$\begin{aligned} \mathcal{B}_T(\omega) &= \sum_{k=1}^n \hat{R}_{1k}(\omega) Y_k^+(\omega) e^{i\omega\Theta} \\ \mathcal{B}_{\bar{T}}(\omega) &= \sum_{k=1}^n \hat{R}_{1k}(\omega) Y_k^-(\omega) e^{-i\omega\Theta} \end{aligned}$$

$$\begin{aligned}
\mathcal{B}_{T^*}(\omega) &= \sum_{k=1}^n \hat{R}_{\mathcal{N}-1k}(\omega) Y_k^+(\omega) e^{i\omega\Theta} \\
\mathcal{B}_{\bar{T}^*}(\omega) &= \sum_{k=1}^n \hat{R}_{\mathcal{N}-1k}(\omega) Y_k^-(\omega) e^{-i\omega\Theta}.
\end{aligned} \tag{62}$$

Taking also into account that

$$\hat{R}_{\mathcal{N}-1k} = \hat{R}_{1\mathcal{N}-k} \quad \text{and} \quad Y_k^+(\lambda) = Y_{\mathcal{N}-k}^-(\lambda), \tag{63}$$

we conclude that

$$\mathcal{B}_{\mathbb{T}}(\omega) = \mathcal{B}_T(\omega) + \mathcal{B}_{\bar{T}}(\omega) + \mathcal{B}_{T^*}(\omega) + \mathcal{B}_{\bar{T}^*}(\omega). \tag{64}$$

The latter leads to the factorization of the defect–particle interaction as described schematically in the figure and in equation (54). The boundary scattering is separated, and as already mentioned is unaffected by the presence of the defect. We identify the following quantities

$$\mathcal{X}(\lambda) = \exp \left[- \int_{-\infty}^{\infty} \frac{d\omega}{\omega} e^{-i\omega\lambda} \mathcal{B}_{\mathcal{X}}(\omega) \right], \quad \mathcal{X} \in \{\mathbb{T}, T, \bar{T}, T^*, \bar{T}^*\}. \tag{65}$$

This concludes our investigation on the scattering amplitudes in the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian.

5 Discussion

We investigate in the present article the generic scattering in the context of the $\mathfrak{sl}(\mathcal{N})$ twisted Yangian. Our analysis is based on the solution of the Bethe ansatz equations in the thermodynamic limit. In particular, in the thermodynamic limit the ground state and low-lying excitations are identified. It is worth emphasizing that in the $\mathfrak{sl}(2n+1)$ case the Bethe ansatz equations are familiar and similar to the $\mathfrak{osp}(1|2n)$ case, whereas in the $\mathfrak{sl}(2n)$ case they are not of any known form and are investigated here for the first time.

The scattering among the particle-like excitations is derived and as expected, turns out to be factorized into a product of the soliton–soliton times the soliton–anti-soliton scattering amplitudes of the bulk $\mathfrak{sl}(\mathcal{N})$ case. We also provide explicit expressions on the boundary scattering amplitudes. We have considered here the simplest boundary matrices i.e. $\mathbb{K}^{\pm} \propto \mathbb{I}$. One of the key points in this investigation together with the study of the boundary scattering is the use of the suitable quantization condition compatible with the underlying algebraic setting as well as the corresponding physical interpretation. The quantization condition on the scattering derived in [5] is clearly confirmed here by the fact that the bulk scattering factorizes into the product of the soliton–soliton and soliton–anti-soliton scattering amplitudes.

Furthermore, we consider the situation where a local integrable defect is inserted. This is achieved by introducing a generic representation of the underlying algebra in a particular

site of the open spin chain. Thus in addition to the bulk and boundary scattering we also investigate the scattering of particle-like excitations with the defect, and derive the associated transmission amplitudes. The key objects here again are the quantization condition together with the derivation of densities of the states in the thermodynamic limit. These lead to the identification of the global transmission amplitude, which turns out to factorize into a product of four distinct terms which describe the soliton–defect, soliton–anti-defect, anti-soliton–defect and anti-soliton–anti-defect interactions of the $\mathfrak{sl}(\mathcal{N})$ spin chain [15].

References

- [1] L. Freidel, J. M. Maillet, Phys. Lett **B262** (1991) 268.
- [2] I.V. Cherednik, Theor. Math. Phys. **61** (1984) 977.
- [3] E.K. Sklyanin, J. Phys. **A21** (1988) 2375.
- [4] G.I. Olshanski “Quantum Groups (1992)”, Springer Lecture notes in Math. 1510
- [5] J. Avan, A. Doikou and N. Karaiskos, arXiv:1410.5991 [hep-th].
- [6] A. Doikou, J. Phys. A **33** (2000) 8797, [hep-th/0006197].
- [7] V. Caudrelier, M. Mintchev, E. Ragoucy and P. Sorba, J.Phys. **A38** (2005) 3431, [hep-th/0412159].
- [8] D. Arnaudon, J. Avan, N. Crampe, A. Doikou, L. Frappat and E. Ragoucy, J. Stat. Mech. **0408** (2004) P08005, [math-ph/0406021].
- [9] D. Arnaudon, J. Avan, N. Crampe, A. Doikou, L. Frappat and E. Ragoucy, Nucl. Phys. B **687** (2004) 257, [math-ph/0310042].
- [10] V. E. Korepin, Commun. Math. Phys. **76** (1980) 165.
- [11] N. Andrei and C. Destri, Nucl. Phys. B **231** (1984) 445.
- [12] M. T. Grisaru, L. Mezincescu and R. I. Nepomechie, J. Phys. A **28** (1995) 1027, [hep-th/9407089].
- [13] B. Sutherland, Phys. Rev. B **12** (1975) 3795;
A. Doikou and R. I. Nepomechie, Nucl. Phys. B **521** (1998) 547 [hep-th/9803118].
- [14] C. N. Yang, Phys. Rev. Lett. **19** (1967) 1312.
- [15] A. Doikou, JHEP 08 (2013) 103, aXiv:1304.5901 [hep-th].